Tables of Radiative Corrections to Pion Nucleon and Nucleon-Nucleon Scattering

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The importance of radiative corrections to hadron scattering is discussed, and the relevant formulas presented in a somewhat simpler form than has been given previously. Tables are given to enable the experimentalist to estimate the numerical importance of the corrections, or evaluate them by interpolation.

Introduction

The scattering of any charged particle is always accompanied by radiation, including both the emission and absorption of virtual photons and the emission of real photons, and a complete analysis of scattering experiments involving charged particles must include corrections for the effects of these photons. These corrections are known as radiative corrections, and they are routinely applied in the analysis of electron and/or muon scattering experiments, according to well-known procedures [1–3]. It seems to be less well known that radiative effects can also be significant for the case of hadron-hadron scattering. In fact, the radiative corrections are numerically important whenever the energy and momentum transfer exceed the rest mass of any particle involved in the scattering process. This means in particular that they should be included in almost all pion scattering experiments, with the possible exception of the Coulomb interference region and at very low energies ($T_\pi < 100$ MeV). This was first pointed out by Sogard [4] who derived an expression for the radiative correction for elastic scattering which is easily modified to include processes of the type $a + b \rightarrow c + d$.

Numerical examples were given which showed that the radiative corrections are rather large (of order of 20%) for large angle scattering at high energies in the case of πp, Kp, and pp scattering. They have also been shown to be non-negligible in the case of pion-nucleus scattering in the region of the (3,3) resonance [5]. Such corrections are not generally made in the analysis of experimental data, although the correct input for phase shift or other analysis is in terms of the radiatively corrected differential cross sections (Radiative corrections to such quantities as polarization parameters are generally small), as has been pointed out by Tromborg and co-workers [6]. Additional electromagnetic effects (notably Coulomb corrections and/or mass differences within an isotopic multiplet) must be taken into account in the more detailed analysis, which is made after radiative effects have been taken into account. These corrections have been studied extensively in [6] and [7]. However, these authors have not given detailed numerical prescriptions for handling the radiative corrections. As an aid to experimentalists, it therefore seems useful to present numerical values for the radiative corrections to πp and pp scattering (elastic and charge exchange) as derived from Sogard's formula, for a range of values of energy and momentum transfer, to indicate when the radiative corrections can be expected to be numerically important. A reasonable estimate of the corrections to be applied can be obtained from the tables by interpolation.

This paper is organized as follows: first a general discussion of the problem of radiative corrections will be given for the case of elastic scattering in which the scattered particle is detected and its momentum measured. By the straightforward application of kinematical identities (see [8]), the expression for the radiative correction derived by Sogard [4] can be somewhat simplified. The simpler (but equivalent) version is given for the case of $\pi^\pm$p elastic scattering, and for the case of charge exchange ($\pi^- p \rightarrow \pi^0 n$). Tables giving numerical values of the radiative correction, assuming a fixed energy resolution ΔE_0 for a large range of values

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of k ($=p_{lab}$) and θ (c.m. scattering angle), as well as the coefficient of $\log(\Delta E_0/\Delta E)$ are given. From these tables it will be possible to estimate the importance of the radiative corrections for a given experimental situation, and, in many cases, to apply them without a great deal of calculation.

The formulas and numerical values given here are derived on the basis of the soft photon approximation, which, for pion-nucleon scattering, is likely to be valid for photon energies less than about 50 MeV (see [9]). For high energies ($k > 1$ GeV/c we have set $\Delta E_0 = \mu$, the pion mass, while for lower incident energies ΔE_0 was set equal to one tenth the lab kinetic energy. In some cases these values are somewhat higher than the expected range of applicability of the soft photon theorem. On the other hand, experimental results on πp bremsstrahlung [9] near the region of the (3.3) resonance indicate that the soft photon approximation works surprisingly well up to $E_\gamma > 100$ MeV, which is comparable to the largest selected value of ΔE_0. The error induced by neglecting possible off-shell effects in hard-photon emission is then expected to be small, and the values of the radiative corrections given in these tables should be fairly reliable.

General Discussion, Kinematics, and Formula for the Radiative Correction to Lowest Order in α

In any experimental arrangement in which the scattered particle is observed, that particle is detected with a certain resolution Δp in momentum and $\Delta\theta$ in angle. It is not possible to distinguish a process in which no radiation has taken place from the same process, accompanied by the emission of one or more low energy photons whose total energy is less than ΔE. Here ΔE is the maximum energy which an undetected photon can have without producing an observable modification of the measured kinematical quantities. Its value is determined by the experimental conditions. If ΔE is not too large, these low energy photons are "soft", which means that their momentum is much less than the relevant momentum transfers involved in the basic process being studied. The probability for emission of such photons is determined only by the gross features of the charge and current distribution and the corresponding matrix element is proportional to the matrix element for the non-radiative process, independent of the spins of the particles being considered, of off-shell effects, or of the interaction dynamics (soft photon theorem). It follows that the soft photon contribution to the radiative correction, which is the dominant one, can be calculated independent of the reaction dynamics. This is not true for the contribution due to so-called "hard" (i.e. not soft) photons; these contributions are normally neglected, at least when the particles involved are hadrons.

We consider a two body scattering process $1+2 \to 3+4$, as shown in diagram 1(a) particle 1 is incident on particle 2, which is at rest in the laboratory. Particle 3 is detected at an angle θ_3 and momentum p_3. Particle 4 is the recoiling target. The other diagrams in Fig. 1(b–g) represent processes involving the emission and reabsorption of a virtual photon and must be added coherently to the diagram 1(a); the interference of the contribution of these diagrams with that of 1(a) produces a correction of order α (α = fine structure constant $\simeq 1/137.04$) to the cross section calculated without allowing for such radiation.

Diagrams 1(b–g) are infrared divergent; that is, they give an infinite contribution as the energy of the virtual photon goes to zero. As mentioned above, for such "soft" virtual photons, the dependence of the matrix element for the basic process on photon momentum or on off shell effects is negligible, and the contribution of the infrared divergent part of these diagrams can be extracted in a model independent way. Other contributions from such graphs cannot be so calculated when the particle involved is a hadron. The infrared divergence is cancelled exactly (to order α) by the contribution to the measured cross section due to the emission of

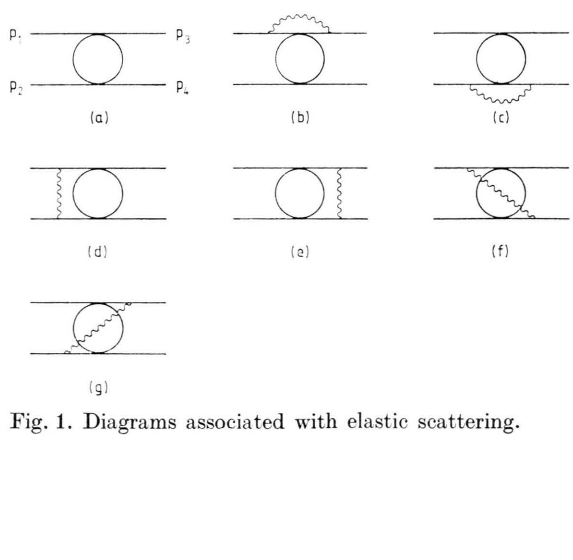

Fig. 1. Diagrams associated with elastic scattering.

undetected (also soft) real photons, as represented by the diagrams of Figure 2. This was first demonstrated semiclassically by Bloch and Nordsieck [10] and later quentum mechanically by (among others) Yennie, Frautschi and Suura [11]. The resulting expression for the cross section is finite, and is given by the product of the nonradiative cross section and a factor whose dominant contribution is known.

To be more specific, Fig. 3 represents the energy spectrum of particle 3 at a fixed scattering angle θ_3. We assume that the width of the peak is much less than the distance between the elastic peak and the threshold for inelastic processes. The tail on the low energy side of the elastic arises from real bremsstrahlung. For practical applications, the measured elastic differential cross section is defined as follows: a cut is made at a distance ΔE below the elastic peak but above the inelastic threshold. After appropriate background subtractions are made, the yield above this cut is integrated. Dividing the integrated yield by the spectrometer solid angle and correcting for detection and identification efficiencies gives the measured differential cross section $\mathrm{d}\sigma_\mathrm{m}/\mathrm{d}\Omega_3$. Events associated with the emission of real photons which are sufficiently low in energy that E_3 is still inside the energy cut are included in the measured cross section; events in which bremsstrahlung has reduced E_3 below the cutoff are excluded. If ΔE is sufficiently small that the factorization implied by the soft photon theorem is a valid approximation, the amount lost from the elastic peak can be determined and a correction factor applied. The radiatively corrected cross section can then be written as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_3} = \frac{\mathrm{d}\sigma_\mathrm{m}}{\mathrm{d}\Omega_3} e^{\delta(\Delta E)} \simeq \frac{\mathrm{d}\sigma_\mathrm{m}}{\mathrm{d}\Omega_3}(1+\delta(\Delta E)) .$$

The exponentiation of δ is justified in [1], [3] and [11].

Sogard [4] has calculated an expression for δ which includes the contribution due to radiation of a real photon of energy less than ΔE as well as the infrared divergent contribution due to emission and reabsorption of virtual photons. Other virtual photon contributions must be neglected in the case of hadrons even though it is known (see Refs. [2], [12]) that for the case of electrons or muons they contribute terms of the same order of magnitude as some which have been kept in the real photon correction. Thus the terms not proportional to $\log(\Delta E)$ in the expression for δ given below are somewhat uncertain. Since the terms involving $\log(\Delta E)$ dominate, this shortcoming is probably not serious. We remark that the authors of Ref. [6] treat these terms (not involving $\log \Delta E$) somewhat differently than is done here, and in [4]. The difference is a matter of definition as to which virtual photon contributions are attributed to radiative corrections and which to other electromagnetic corrections. For most applications the difference is numerically not important. In this work only the infrared divergent virtual photon terms were kept in the radiative correction.

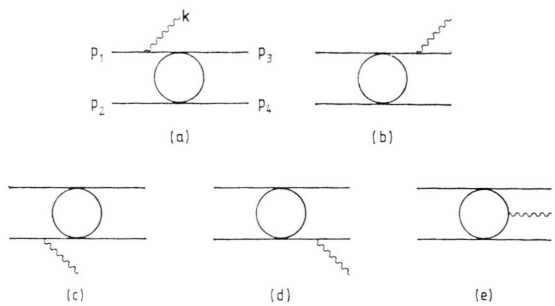

Fig. 2. Diagrams associated with the emission of a real photon. For sufficiently low energy photons, the contribution of diagram 2(e) is negligible.

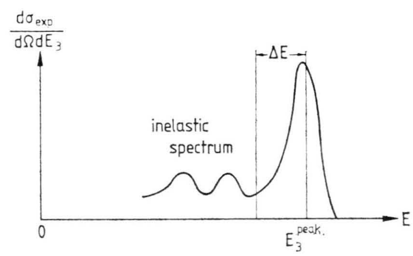

Fig. 3. The elastic peak at fixed scattering angle together with the tail arising from real bremsstrahlung and the onset of the inelastic threshold. Effects of experimental resolution are also shown.

For elastic scattering, Sogard obtains

$$\delta_\pm \simeq \frac{\alpha}{\pi}\left\{\left[\frac{1}{2v_{13}}\ln\left(\frac{1+v_{13}}{1-v_{13}}\right)-1\right]\ln\frac{\omega\,\omega_\mathrm{f}}{(\Delta E)^2}+\frac{1}{2v_{13}}\ln\left(\frac{1+v_{13}}{1-v_{13}}\right)\ln\left(1+\frac{\mu^2 t}{4m^2\,\omega\,\omega_\mathrm{f}}\right)\right.$$
$$\left.+\left[\frac{1}{2\beta_4}\ln\left(\frac{1+\beta_4}{1-\beta_4}\right)-1\right]\ln\frac{m\,\omega_\mathrm{N}}{(\Delta E)^2}+\frac{1}{2\beta_4}\ln\left(\frac{1+\beta_4}{1-\beta_4}\right)\ln\left(\frac{1-t/4m^2}{1-t/2m^2}\right)\right\}$$

$$+\frac{1}{v_{13}}\left[L_2\left(\frac{Q+t}{Q+2m(\omega+\omega_f)}\right)+L_2\left(\frac{Q+t}{Q-2m(\omega+\omega_f)}\right)\right.$$
$$\left.-L_2\left(\frac{Q-t}{Q+2m(\omega+\omega_f)}\right)-L_2\left(\frac{Q-t}{Q-2m(\omega+\omega_f)}\right)\right]$$
$$+\frac{1}{\beta_4}\left[L_2\left(\frac{t}{Q'}\right)-L_2\left(-\frac{t}{Q'}\right)+L_2\left(-\frac{t}{Q'}\,\frac{Q'+t}{Q'-t}\right)-L_2\left(\frac{t}{Q'}\,\frac{Q'-t}{Q'+t}\right)\right]$$
$$\left.\pm\left(\frac{B_1+B_2}{\beta_3}-\frac{A_1+A_2}{\beta_1}\right)\right\}$$

and for charge exchange

$$\delta_0=\frac{\alpha}{\pi}\left[\frac{A_1}{\beta_1}-\ln\frac{\omega_f\,\omega_N}{(\Delta E)^2}\right].$$

The notation for kinematics follows Reference [8]. The most important relations for the laboratory frame are given by (k = lab momentum, ω = lab total energy, q = c.m. momentum)

$$2m\,\omega=s-m^2-\mu^2,\quad 2m\,\omega_f=2m\,\omega+t;\quad 2m\,\omega_N=2m^2-t;$$
$$\beta_1=\frac{k}{\omega}=\frac{q\sqrt{s}}{m\,\omega};\quad \beta_3=\frac{k_f}{\omega_f};$$
$$\beta_4=[t(t-4m]^2)^{1/2}/(2m^2-t)=Q'/(2m^2-t);\quad v_{13}=[t(t-4\mu^2)]^{1/2}/(2\mu^2-t)=Q/(2\mu^2-t);$$
$$t=\frac{-2m\,k^2[m-m\cos\theta_3(1-\mu^2\sin^2\theta_3/m^2)^{1/2}+\omega\sin^2\theta_3]}{(\omega+m)^2-k^2\cos\theta_3}.$$

Further

$$A_1=\ln\left(\frac{1+\beta_1}{1-\beta_1}\right)\ln\frac{\omega_N}{\Delta E}+\ln\frac{\omega(1+\beta_1)}{m}\ln\left|\frac{m\,\omega_f-\omega\,\omega_N(1+\beta_1)}{m\,\omega_N-\omega\,\omega_N(1+\beta_1)}\right|$$
$$-\ln\frac{\omega(1-\beta_1)}{m}\ln\left|\frac{m\,\omega_f-\omega\,\omega_N(1-\beta_1)}{m\,\omega_N-\omega\,\omega_N(1-\beta_1)}\right|$$
$$+L_2\left(\frac{m(\omega_f-\omega_N)}{m\,\omega_f-\omega\,\omega_N(1+\beta_1)}\right)-L_2\left(\frac{\omega(1+\beta_1)(\omega_f-\omega_N)}{m\,\omega_f-\omega\,\omega_N(1+\beta_1)}\right)$$
$$-L_2\left(\frac{m(\omega_f-\omega_N)}{m\,\omega_f-\omega\,\omega_N(1-\beta_1)}\right)+L_2\left(\frac{\omega(1-\beta_1)(\omega_f-\omega_N)}{m\,\omega_f-\omega\,\omega_N(1-\beta_1)}\right);$$

$$A_2=\ln\left(\frac{1+\beta_1}{1-\beta_1}\right)\ln\frac{k}{\Delta E}+\ln\frac{\omega(1-\beta_1)}{m}\ln\left|1-\frac{\omega(1-\beta_1)}{m}\right|$$
$$-\ln\frac{\omega(1+\beta_1)}{m}\ln\left|1-\frac{\omega(1+\beta_1)}{m}\right|+L_2\left(\frac{m-\omega}{\beta_1\omega}\right)$$
$$-L_2\left(\frac{\omega-m}{\beta_1\omega}\right)+L_2\left(\frac{\omega-m}{m}\,\frac{1-\beta_1}{\beta_1}\right)-L_2\left(\frac{m-\omega}{m}\,\frac{1+\beta_1}{\beta_1}\right);$$

$$B_1=\ln\left(\frac{1+\beta_3}{1-\beta_3}\right)\ln\frac{k_f}{\Delta E}+\ln\frac{\omega_f(1-\beta_3)}{m}\ln\left|1-\frac{\omega_f(1-\beta_3)}{m}\right|$$
$$-\ln\frac{\omega_f(1+\beta_3)}{m}\ln\left|1-\frac{\omega_f(1+\beta_3)}{m}\right|+L_2\left(\frac{m-\omega_f}{\beta_3\omega_f}\right)$$
$$-L_2\left(\frac{\omega_f-m}{\beta_3\omega_f}\right)+L_2\left(\frac{\omega_f-m}{m}\,\frac{1-\beta_3}{\beta_3}\right)-L_2\left(\frac{m-\omega_f}{m}\,\frac{1+\beta_3}{\beta_3}\right);$$

Table 1. Radiative corrections (in percent) for elastic pion-nucleon scattering. See text for full description.

PLAB	W	Θ	$\|T\|$	100 δ_+	100 δ_-	100 δ_0	100 C_+	100 C_-	100 C_0
0.2	1.165	10.0	0.001	0.0156	0.0249	0.5657	0.0051	0.0076	0.1908
		20.0	0.003	0.0606	0.0973	0.5650	0.0197	0.0299	0.1908
		30.0	0.007	0.1295	0.2114	0.5638	0.0423	0.0650	0.1908
		40.0	0.012	0.2153	0.3587	0.5621	0.0706	0.1104	0.1908
		50.0	0.019	0.3100	0.5300	0.5601	0.1021	0.1632	0.1908
		60.0	0.026	0.4066	0.7161	0.5578	0.1347	0.2207	0.1908
		70.0	0.034	0.4994	0.9091	0.5553	0.1664	0.2804	0.1908
		80.0	0.043	0.5845	1.1021	0.5525	0.1960	0.3403	0.1908
		90.0	0.052	0.6592	1.2897	0.5497	0.2227	0.3987	0.1908
		100.0	0.061	0.7227	1.4675	0.5469	0.2460	0.4542	0.1908
		110.0	0.070	0.7747	1.6318	0.5442	0.2657	0.5056	0.1908
		120.0	0.078	0.8159	1.7797	0.5417	0.2820	0.5521	0.1908
		130.0	0.085	0.8475	1.9088	0.5394	0.2950	0.5928	0.1908
		140.0	0.092	0.8707	2.0172	0.5375	0.3050	0.6270	0.1908
		150.0	0.097	0.8870	2.1031	0.5359	0.3124	0.6543	0.1908
		160.0	0.101	0.8977	2.1654	0.5347	0.3174	0.6741	0.1908
		170.0	0.103	0.9036	2.2031	0.5340	0.3203	0.6861	0.1908
		180.0	0.104	0.9055	2.2158	0.5338	0.3213	0.6901	0.1908
0.4	1.302	10.0	0.003	0.0469	0.0625	1.1174	0.0175	0.0228	0.4088
		20.0	0.010	0.1747	0.2369	1.1144	0.0653	0.0863	0.4088
		30.0	0.022	0.3526	0.4918	1.1095	0.1323	0.1794	0.4088
		40.0	0.039	0.5480	0.7928	1.1027	0.2068	0.2897	0.4088
		50.0	0.059	0.7359	1.1134	1.0943	0.2797	0.4077	0.4088
		60.0	0.083	0.9013	1.4358	1.0845	0.3455	0.5272	0.4088
		70.0	0.109	1.0373	1.7496	1.0736	0.4017	0.6444	0.4088
		80.0	0.137	1.1417	2.0489	1.0617	0.4473	0.7573	0.4088
		90.0	0.166	1.2158	2.3298	1.0493	0.4825	0.8644	0.4088
		100.0	0.195	1.2627	2.5898	1.0366	0.5082	0.9647	0.4088
		110.0	0.223	1.2865	2.8267	1.0242	0.5256	1.0573	0.4088
		120.0	0.249	1.2922	3.0383	1.0123	0.5362	1.1410	0.4088
		130.0	0.273	1.2850	3.2224	1.0014	0.5414	1.2147	0.4088
		140.0	0.294	1.2700	3.3769	0.9919	0.5428	1.2773	0.4088
		150.0	0.310	1.2523	3.4997	0.9840	0.5420	1.3276	0.4088
		160.0	0.322	1.2362	3.5889	0.9782	0.5404	1.3644	0.4088
		170.0	0.330	1.2251	3.6430	0.9746	0.5389	1.3869	0.4088
		180.0	0.332	1.2211	3.6612	0.9734	0.5384	1.3944	0.4088
0.6	1.434	10.0	0.005	0.0849	0.1039	1.4714	0.0332	0.0403	0.5679
		20.0	0.019	0.3031	0.3788	1.4660	0.1190	0.1471	0.5679
		30.0	0.041	0.5804	0.7501	1.4571	0.2289	0.2920	0.5679
		40.0	0.072	0.8560	1.1556	1.4450	0.3398	0.4512	0.5679
		50.0	0.110	1.0969	1.5608	1.4298	0.4390	0.6118	0.5679
		60.0	0.154	1.2904	1.9504	1.4119	0.5216	0.7681	0.5679
		70.0	0.203	1.4340	2.3186	1.3917	0.5866	0.9181	0.5679
		80.0	0.255	1.5302	2.6639	1.3697	0.6347	1.0611	0.5679
		90.0	0.308	1.5839	2.9855	1.3463	0.6674	1.1968	0.5679
		100.0	0.362	1.6011	3.2825	1.3222	0.6867	1.3248	0.5679
		110.0	0.414	1.5885	3.5539	1.2981	0.6945	1.4442	0.5679
		120.0	0.462	1.5536	3.7975	1.2747	0.6933	1.5538	0.5679
		130.0	0.507	1.5045	4.0111	1.2529	0.6856	1.6520	0.5679
		140.0	0.545	1.4492	4.1915	1.2335	0.6741	1.7366	0.5679
		150.0	0.575	1.3958	4.3360	1.2173	0.6615	1.8057	0.5679
		160.0	0.598	1.3517	4.4415	1.2051	0.6504	1.8569	0.5679
		170.0	0.612	1.3226	4.5059	1.1975	0.6427	1.8885	0.5679
		180.0	0.617	1.3125	4.5275	1.1950	0.6400	1.8991	0.5679
0.8	1.557	10.0	0.007	0.1254	0.1463	1.7243	0.0504	0.0586	0.6891
		20.0	0.028	0.4301	0.5136	1.7168	0.1733	0.2062	0.6891
		30.0	0.062	0.7878	0.9752	1.7044	0.3190	0.3929	0.6891
		40.0	0.109	1.1177	1.4497	1.6875	0.4557	0.5868	0.6891
		50.0	0.166	1.3887	1.9043	1.6662	0.5712	0.7751	0.6891
		60.0	0.232	1.5939	2.3305	1.6411	0.6628	0.9547	0.6891
		70.0	0.306	1.7360	2.7280	1.6126	0.7314	1.1257	0.6891
		80.0	0.384	1.8207	3.0985	1.5812	0.7791	1.2888	0.6891
		90.0	0.465	1.8550	3.4433	1.5477	0.8079	1.4444	0.6891
		100.0	0.546	1.8461	3.7627	1.5127	0.8203	1.5926	0.6891
		110.0	0.624	1.8023	4.0560	1.4773	0.8188	1.7327	0.6891
		120.0	0.697	1.7325	4.3210	1.4424	0.8063	1.8630	0.6891
		130.0	0.764	1.6467	4.5549	1.4094	0.7861	1.9816	0.6891
		140.0	0.821	1.5555	4.7540	1.3797	0.7616	2.0855	0.6891
		150.0	0.868	1.4697	4.9143	1.3545	0.7369	2.1715	0.6891
		160.0	0.902	1.3995	5.0321	1.3353	0.7158	2.2361	0.6891
		170.0	0.923	1.3536	5.1043	1.3233	0.7016	2.2763	0.6891
		180.0	0.930	1.3376	5.1286	1.3192	0.6966	2.2900	0.6891

Table 1 (continued)

PLAB	W	Θ	|T|	100 δ_+	100 δ_-	100 δ_0	100 C_+	100 C_-	100 C_0
1.0	1.672	10.0	0.010	0.1345	0.1523	1.5465	0.0681	0.0772	0.7864
		20.0	0.038	0.4438	0.5149	1.5372	0.2255	0.2619	0.7864
		30.0	0.084	0.7824	0.9422	1.5219	0.4001	0.4819	0.7864
		40.0	0.147	1.0765	1.3601	1.5009	0.5555	0.7007	0.7864
		50.0	0.225	1.3060	1.7478	1.4745	0.6819	0.9084	0.7864
		60.0	0.315	1.4708	2.1038	1.4432	0.7793	1.1044	0.7864
		70.0	0.415	1.5761	2.4314	1.4076	0.8500	1.2906	0.7864
		80.0	0.521	1.6285	2.7339	1.3681	0.8967	1.4686	0.7864
		90.0	0.630	1.6345	3.0134	1.3255	0.9221	1.6396	0.7864
		100.0	0.739	1.6011	3.2708	1.2808	0.9286	1.8039	0.7864
		110.0	0.846	1.5358	3.5056	1.2350	0.9191	1.9607	0.7864
		120.0	0.945	1.4470	3.7166	1.1894	0.8966	2.1085	0.7864
		130.0	1.035	1.3444	3.9014	1.1456	0.8649	2.2447	0.7864
		140.0	1.113	1.2385	4.0576	1.1055	0.8283	2.3659	0.7864
		150.0	1.176	1.1404	4.1825	1.0712	0.7918	2.4675	0.7864
		160.0	1.222	1.0608	4.2735	1.0448	0.7608	2.5448	0.7864
		170.0	1.251	1.0089	4.3289	1.0281	0.7399	2.5934	0.7864
		180.0	1.260	0.9909	4.3475	1.0224	0.7325	2.6100	0.7864
2.0	2.159	10.0	0.023	0.4093	0.4363	2.8132	0.1544	0.1656	1.0985
		20.0	0.091	1.1659	1.2743	2.7982	0.4418	0.4868	1.0985
		30.0	0.202	1.8360	2.0815	2.7737	0.7006	0.8021	1.0985
		40.0	0.353	2.3476	2.7873	2.7400	0.9042	1.0852	1.0985
		50.0	0.540	2.7183	3.4106	2.6978	1.0589	1.3428	1.0985
		60.0	0.755	2.9701	3.9748	2.6473	1.1725	1.5831	1.0985
		70.0	0.994	3.1193	4.4962	2.5890	1.2506	1.8123	1.0985
		80.0	1.248	3.1773	4.9849	2.5232	1.2970	2.0342	1.0985
		90.0	1.511	3.1531	5.4464	2.4504	1.3142	2.2513	1.0985
		100.0	1.773	3.0549	5.8831	2.3713	1.3042	2.4650	1.0985
		110.0	2.027	2.8920	6.2945	2.2869	1.2693	2.6757	1.0985
		120.0	2.266	2.6763	6.6777	2.1990	1.2122	2.8821	1.0985
		130.0	2.482	2.4234	7.0274	2.1102	1.1373	3.0815	1.0985
		140.0	2.668	2.1544	7.3356	2.0243	1.0510	3.2684	1.0985
		150.0	2.819	1.8957	7.5924	1.9467	0.9626	3.4343	1.0985
		160.0	2.930	1.6779	7.7867	1.8838	0.8843	3.5675	1.0985
		170.0	2.998	1.5317	7.9081	1.8424	0.8298	3.6548	1.0985
		180.0	3.021	1.4800	7.9494	1.8279	0.8102	3.6853	1.0985
4.0	2.900	10.0	0.051	0.9960	1.0317	4.4553	0.2999	0.3125	1.4174
		20.0	0.202	2.3897	2.5337	4.4365	0.7254	0.7758	1.4174
		30.0	0.449	3.4291	3.7578	4.4061	1.0518	1.1658	1.4174
		40.0	0.784	4.1767	4.7710	4.3649	1.2959	1.4999	1.4174
		50.0	1.197	4.7110	5.6570	4.3131	1.4796	1.8007	1.4174
		60.0	1.675	5.0751	6.4640	4.2505	1.6146	2.0813	1.4174
		70.0	2.204	5.2919	7.2185	4.1764	1.7073	2.3495	1.4174
		80.0	2.768	5.3735	7.9355	4.0902	1.7608	2.6102	1.4174
		90.0	3.350	5.3272	8.6239	3.9911	1.7770	2.8675	1.4174
		100.0	3.931	5.1589	9.2880	3.8789	1.7570	3.1241	1.4174
		110.0	4.495	4.8750	9.9297	3.7536	1.7015	3.3826	1.4174
		120.0	5.024	4.4856	10.5473	3.6162	1.6115	3.6443	1.4174
		130.0	5.503	4.0071	11.1354	3.4690	1.4893	3.9088	1.4174
		140.0	5.916	3.4675	11.6821	3.3170	1.3404	4.1721	1.4174
		150.0	6.250	2.9126	12.1666	3.1691	1.1765	4.4240	1.4174
		160.0	6.497	2.4120	12.5576	3.0397	1.0190	4.6437	1.4174
		170.0	6.648	2.0546	12.8158	2.9486	0.9007	4.7990	1.4174
		180.0	6.699	1.9236	12.9066	2.9154	0.8561	4.8560	1.4174
6.0	3.487	10.0	0.079	1.5413	1.5804	5.5845	0.4157	0.4285	1.6051
		20.0	0.314	3.3528	3.5151	5.5656	0.9147	0.9669	1.6051
		30.0	0.698	4.6158	4.9896	5.5356	1.2761	1.3946	1.6051
		40.0	1.219	5.5167	6.1976	5.4951	1.5452	1.7576	1.6051
		50.0	1.862	6.1665	7.2571	5.4438	1.7489	2.0838	1.6051
		60.0	2.606	6.6170	8.2263	5.3801	1.8995	2.3870	1.6051
		70.0	3.429	6.8909	9.1366	5.3025	2.0030	2.6758	1.6051
		80.0	4.307	7.0006	10.0042	5.2094	2.0630	2.9558	1.6051
		90.0	5.212	6.9517	10.8402	5.0993	2.0812	3.2317	1.6051
		100.0	6.117	6.7475	11.6514	4.9710	2.0581	3.5078	1.6051
		110.0	6.994	6.3918	12.4425	4.8236	1.9938	3.7881	1.6051
		120.0	7.818	5.8904	13.2153	4.6572	1.8879	4.0760	1.6051
		130.0	8.562	5.2556	13.9675	4.4731	1.7406	4.3736	1.6051
		140.0	9.204	4.5130	14.6893	4.2754	1.5545	4.6802	1.6051
		150.0	9.725	3.7138	15.3572	4.0738	1.3394	4.9878	1.6051
		160.0	10.109	2.9530	15.9248	3.8877	1.1197	5.2727	1.6051
		170.0	10.345	2.3803	16.3195	3.7494	0.9437	5.4876	1.6051
		180.0	10.424	2.1630	16.4629	3.6971	0.8742	5.5699	1.6051

Table 1 (continued)

PLAB	W	Θ	$\|T\|$	100 δ_+	100 δ_-	100 δ_0	100 C_+	100 C_-	100 C_0
10.0	4.435	10.0	0.136	2.4796	2.5229	7.1810	0.5922	0.6052	1.8420
		20.0	0.540	4.8327	5.0135	7.1658	1.1746	1.2276	1.8420
		30.0	1.199	6.3921	6.8203	7.1430	1.5805	1.7027	1.8420
		40.0	2.095	7.5132	8.3002	7.1116	1.8845	2.1039	1.8420
		50.0	3.198	8.3370	9.6044	7.0688	2.1156	2.4617	1.8420
		60.0	4.476	8.9182	10.8014	7.0112	2.2862	2.7914	1.8420
		70.0	5.891	9.2823	11.9245	6.9356	2.4035	3.1024	1.8420
		80.0	7.398	9.4412	12.9934	6.8393	2.4717	3.4016	1.8420
		90.0	8.953	9.3988	14.0233	6.7199	2.4929	3.6956	1.8420
		100.0	10.507	9.1576	15.0246	6.5753	2.4679	3.9898	1.8420
		110.0	12.015	8.7151	16.0090	6.4034	2.3954	4.2906	1.8420
		120.0	13.429	8.0722	16.9833	6.2026	2.2740	4.6034	1.8420
		130.0	14.708	7.2281	17.9517	5.9720	2.0999	4.9338	1.8420
		140.0	15.811	6.2013	18.9177	5.7134	1.8722	5.2876	1.8420
		150.0	16.706	5.0312	19.8616	5.4345	1.5929	5.6633	1.8420
		160.0	17.366	3.8301	20.7269	5.1575	1.2827	6.0414	1.8420
		170.0	17.770	2.8448	21.3835	4.9338	1.0076	6.3569	1.8420
		180.0	17.906	2.4464	21.6380	4.8435	0.8902	6.4877	1.8420
50.0	9.733	10.0	0.706	7.4350	7.4953	13.4499	1.3342	1.3482	2.5873
		20.0	2.802	11.6050	11.8507	13.5139	2.1669	2.2216	2.5873
		30.0	6.226	14.3618	14.9876	13.5945	2.7315	2.8626	2.5873
		40.0	10.872	16.4343	17.5331	13.6620	3.1457	3.3698	2.5873
		50.0	16.599	17.9525	19.7628	13.7010	3.4436	3.8013	2.5873
		60.0	23.235	19.0462	21.7597	13.7042	3.6577	4.1816	2.5873
		70.0	30.576	19.7642	23.5835	13.6675	3.8032	4.5280	2.5873
		80.0	38.400	20.1126	25.3047	13.5880	3.8852	4.8577	2.5873
		90.0	46.469	20.1321	26.9327	13.4629	3.9132	5.1761	2.5873
		100.0	54.538	19.8098	28.5147	13.2885	3.8861	5.4951	2.5873
		110.0	62.363	19.1337	30.0837	13.0600	3.8020	5.8242	2.5873
		120.0	69.704	18.0928	31.6611	12.7710	3.6582	6.1719	2.5873
		130.0	76.339	16.6351	33.3009	12.4127	3.4427	6.5542	2.5873
		140.0	82.067	14.7078	35.0470	11.9728	3.1409	6.9888	2.5873
		150.0	86.713	12.2227	36.9702	11.4360	2.7259	7.5050	2.5873
		160.0	90.136	9.0869	39.1522	10.7898	2.1567	8.1454	2.5873
		170.0	92.232	5.4589	41.5334	10.0758	1.4180	8.9264	2.5873
		180.0	92.938	3.2813	42.9320	9.6605	0.9184	9.4400	2.5873
20.0	6.199	10.0	0.278	4.2344	4.2851	9.6529	0.8798	0.8933	2.1637
		20.0	1.105	7.3293	7.5429	9.6550	1.5655	1.6206	2.1637
		30.0	2.455	9.3510	9.8529	9.6594	2.0394	2.1646	2.1637
		40.0	4.288	10.8322	11.7618	9.6577	2.3936	2.6184	2.1637
		50.0	6.546	11.9357	13.4469	9.6413	2.6594	3.0154	2.1637
		60.0	9.163	12.7267	14.9821	9.6037	2.8536	3.3737	2.1637
		70.0	12.058	13.2401	16.4045	9.5402	2.9871	3.7051	2.1637
		80.0	15.144	13.4758	17.7527	9.4475	3.0630	4.0215	2.1637
		90.0	18.326	13.4553	19.0473	9.3223	3.0878	4.3310	2.1637
		100.0	21.508	13.1735	20.3061	9.1617	3.0613	4.6408	2.1637
		110.0	24.594	12.6240	21.5493	8.9621	2.9820	4.9588	2.1637
		120.0	27.489	11.7979	22.7940	8.7196	2.8467	5.2933	2.1637
		130.0	30.106	10.6792	24.0612	8.4299	2.6487	5.6546	2.1637
		140.0	32.365	9.2525	25.3707	8.0892	2.3787	6.0550	2.1637
		150.0	34.197	7.5163	26.7346	7.6972	2.0258	6.5073	2.1637
		160.0	35.547	5.5436	28.1229	7.2694	1.5892	7.0139	2.1637
		170.0	36.374	3.6746	29.3480	6.8743	1.1329	7.5118	2.1637
		180.0	36.652	2.8118	29.8930	6.6934	0.9048	7.7536	2.1637
100.0	13.732	10.0	1.419	10.5999	10.5745	16.7008	1.7436	1.7436	2.9024
		20.0	5.632	15.6877	15.9523	16.8563	2.6907	2.7439	2.9024
		30.0	12.511	19.1876	19.7105	17.0245	3.3260	3.4294	2.9024
		40.0	21.847	21.7193	22.7781	17.1607	3.7667	3.9631	2.9024
		50.0	33.357	23.4728	25.5185	17.2510	4.0649	4.4233	2.9024
		60.0	46.691	24.8838	27.7651	17.2910	4.3009	4.7971	2.9024
		70.0	61.444	25.6725	29.9626	17.2790	4.4383	5.1608	2.9024
		80.0	77.166	26.1279	31.9261	17.2135	4.5261	5.4905	2.9024
		90.0	93.382	26.2099	33.7654	17.0921	4.5605	5.8066	2.9024
		100.0	109.598	25.8263	35.5697	16.9113	4.5277	6.1263	2.9024
		110.0	125.321	25.0666	37.3756	16.6657	4.4427	6.4595	2.9024
		120.0	140.073	23.8776	39.1970	16.3472	4.2963	6.8122	2.9024
		130.0	153.407	22.1947	41.0955	15.9442	4.0762	7.2007	2.9024
		140.0	164.917	19.9240	43.1491	15.4389	3.7632	7.6478	2.9024
		150.0	174.253	16.9153	45.4694	14.8044	3.3236	8.1895	2.9024
		160.0	181.133	12.9194	48.2413	14.0017	2.6927	8.8921	2.9024
		170.0	185.346	7.6800	51.6939	13.0100	1.7624	9.8651	2.9024
		180.0	186.764	3.6555	54.3301	12.2932	0.9335	10.7080	2.9024

Table 2. Radiative corrections (in percent) for nucleon-nucleon scattering. See text for full description.

PLAB	W	Θ	$\|T\|$	100 δ_+	100 δ_-	100 δ_0	100 C_+	100 C_-	100 C_0
1.0	2.082	10.0	0.006	0.0010	0.0081	0.2708	0.0005	0.0039	0.1260
		20.0	0.025	0.0038	0.0322	0.2704	0.0018	0.0153	0.1260
		30.0	0.054	0.0080	0.0716	0.2698	0.0039	0.0340	0.1260
		40.0	0.095	0.0133	0.1252	0.2690	0.0064	0.0594	0.1260
		50.0	0.145	0.0189	0.1912	0.2682	0.0091	0.0905	0.1260
		60.0	0.203	0.0241	0.2678	0.2675	0.0117	0.1265	0.1260
		70.0	0.267	0.0284	0.3528	0.2669	0.0137	0.1662	0.1260
		80.0	0.336	0.0312	0.4435	0.2665	0.0151	0.2085	0.1260
		90.0	0.406	0.0321	0.5373	0.2663	0.0155	0.2521	0.1260
		100.0	0.477	0.0312	0.6313	0.2665	0.0151	0.2957	0.1260
		110.0	0.545	0.0284	0.7228	0.2669	0.0137	0.3379	0.1260
		120.0	0.609	0.0241	0.8090	0.2675	0.0117	0.3777	0.1260
		130.0	0.667	0.0189	0.8871	0.2682	0.0091	0.4137	0.1260
		140.0	0.717	0.0133	0.9548	0.2690	0.0064	0.4448	0.1260
		150.0	0.758	0.0080	1.0098	0.2698	0.0039	0.4702	0.1260
		160.0	0.788	0.0038	1.0504	0.2704	0.0018	0.4889	0.1260
		170.0	0.806	0.0010	1.0754	0.2708	0.0005	0.5003	0.1260
6.0	3.627	10.0	0.073	0.0552	0.0898	2.3688	0.0197	0.0312	0.7366
		20.0	0.291	0.2108	0.3507	2.3670	0.0744	0.1205	0.7366
		30.0	0.645	0.4400	0.7600	2.3658	0.1531	0.2571	0.7366
		40.0	1.127	0.7068	1.2869	2.3659	0.2423	0.4278	0.7366
		50.0	1.721	0.9750	1.8996	2.3675	0.3296	0.6205	0.7366
		60.0	2.409	1.2135	2.5706	2.3698	0.4054	0.8263	0.7366
		70.0	3.170	1.3992	3.2784	2.3721	0.4634	1.0392	0.7366
		80.0	3.981	1.5165	4.0068	2.3738	0.4995	1.2555	0.7366
		90.0	4.818	1.5566	4.7440	2.3744	0.5117	1.4733	0.7366
		100.0	5.654	1.5165	5.4799	2.3738	0.4995	1.6910	0.7366
		110.0	6.466	1.3992	6.2050	2.3721	0.4634	1.9074	0.7366
		120.0	7.227	1.2135	6.9081	2.3698	0.4054	2.1202	0.7366
		130.0	7.915	0.9750	7.5745	2.3675	0.3296	2.3260	0.7366
		140.0	8.508	0.7068	8.1841	2.3659	0.2423	2.5188	0.7366
		150.0	8.990	0.4400	8.7107	2.3658	0.1531	2.6895	0.7366
		160.0	9.345	0.2108	9.1225	2.3670	0.0744	2.8261	0.7366
		170.0	9.562	0.0552	9.3869	2.3688	0.0197	2.9154	0.7366
10.0	4.540	10.0	0.130	0.1181	0.1599	3.4710	0.0384	0.0510	0.9640
		20.0	0.515	0.4433	0.6136	3.4738	0.1417	0.1922	0.9640
		30.0	1.145	0.9027	1.2958	3.4811	0.2824	0.3966	0.9640
		40.0	1.999	1.4119	2.1307	3.4927	0.4323	0.6364	0.9640
		50.0	3.052	1.8998	3.0551	3.5063	0.5708	0.8921	0.9640
		60.0	4.272	2.3167	4.0257	3.5195	0.6856	1.1525	0.9640
		70.0	5.622	2.6315	5.0168	3.5303	0.7703	1.4125	0.9640
		80.0	7.060	2.8264	6.0140	3.5374	0.8220	1.6708	0.9640
		90.0	8.544	2.8922	7.0103	3.5398	0.8393	1.9279	0.9640
		100.0	10.027	2.8264	8.0018	3.5374	0.8220	2.1850	0.9640
		110.0	11.466	2.6315	8.9850	3.5303	0.7703	2.4433	0.9640
		120.0	12.816	2.3167	9.9544	3.5195	0.6856	2.7034	0.9640
		130.0	14.036	1.8998	10.8986	3.5063	0.5708	2.9637	0.9640
		140.0	15.089	1.4119	11.7957	3.4927	0.4323	3.2194	0.9640
		150.0	15.943	0.9027	12.6076	3.4811	0.2824	3.4593	0.9640
		160.0	16.572	0.4433	13.2751	3.4738	0.1417	3.6637	0.9640
		170.0	16.958	0.1181	13.7231	3.4710	0.0384	3.8049	0.9640
20.0	6.272	10.0	0.272	0.2935	0.3432	5.2983	0.0848	0.0982	1.2809
		20.0	1.080	1.0541	1.2609	5.3190	0.2961	0.3499	1.2809
		30.0	2.399	2.0336	2.5168	5.3548	0.5531	0.6749	1.2809
		40.0	4.189	3.0214	3.9136	5.3983	0.7984	1.0165	1.2809
		50.0	6.396	3.8968	5.3420	5.4415	1.0059	1.3501	1.2809
		60.0	8.953	4.6025	6.7550	5.4790	1.1673	1.6691	1.2809
		70.0	11.782	5.1149	8.1381	5.5076	1.2815	1.9747	1.2809
		80.0	14.797	5.4243	9.4920	5.5254	1.3493	2.2707	1.2809
		90.0	17.906	5.5277	10.8240	5.5314	1.3718	2.5617	1.2809
		100.0	21.016	5.4243	12.1440	5.5254	1.3493	2.8528	1.2809
		110.0	24.031	5.1148	13.4624	5.5076	1.2815	3.1487	1.2809
		120.0	26.859	4.6025	14.7884	5.4790	1.1673	3.4543	1.2809
		130.0	29.416	3.8966	16.1262	5.4415	1.0059	3.7733	1.2809
		140.0	31.623	3.0213	17.4682	5.3983	0.7983	4.1070	1.2809
		150.0	33.413	2.0336	18.7783	5.3549	0.5531	4.4486	1.2809
		160.0	34.733	1.0541	19.9626	5.3191	0.2961	4.7735	1.2809
		170.0	35.540	0.2934	20.8387	5.2983	0.0848	5.0253	1.2809

Table 2 (continued)

PLAB	W	Θ	$\|T\|$	100 δ_+	100 δ_-	100 δ_0	100 C_+	100 C_-	100 C_0
40.0	8.766	10.0	0.557	0.6655	0.7216	7.4930	0.1725	0.1861	1.6013
		20.0	2.211	2.2164	2.4536	7.5554	0.5500	0.6048	1.6013
		30.0	4.912	3.9573	4.5241	7.6454	0.9420	1.0672	1.6013
		40.0	8.577	5.5334	6.5878	7.7399	1.2742	1.4988	1.6013
		50.0	13.096	6.8308	8.5476	7.8253	1.5341	1.8893	1.6013
		60.0	18.331	7.8299	10.3936	7.8951	1.7270	2.2455	1.6013
		70.0	24.123	8.5339	12.1451	7.9461	1.8594	2.5774	1.6013
		80.0	30.296	8.9520	13.8260	7.9771	1.9368	2.8937	1.6013
		90.0	36.662	9.0907	15.4613	7.9875	1.9622	3.2026	1.6013
		100.0	43.028	8.9519	17.0760	7.9771	1.9368	3.5114	1.6013
		110.0	49.201	8.5338	18.6949	7.9461	1.8594	3.8278	1.6013
		120.0	54.993	7.8296	20.3442	7.8951	1.7269	4.1596	1.6013
		130.0	60.228	6.8308	22.0510	7.8254	1.5341	4.5158	1.6013
		140.0	64.747	5.5328	23.8395	7.7400	1.2741	4.9063	1.6013
		150.0	68.412	3.9569	25.7143	7.6455	0.9420	5.3379	1.6013
		160.0	71.113	2.2151	27.6040	7.5555	0.5498	5.8002	1.6013
		170.0	72.767	0.6652	29.2123	7.4932	0.1725	6.2190	1.6013
100.0	13.763	10.0	1.412	1.7606	1.8253	10.9542	0.3978	0.4116	2.0263
		20.0	5.606	5.0060	5.2898	11.1290	1.0615	1.1176	2.0263
		30.0	12.453	7.9586	8.6332	11.3301	1.6084	1.7351	2.0263
		40.0	21.747	10.3232	11.5820	11.5136	2.0163	2.2437	2.0263
		50.0	33.204	12.1415	14.2002	11.6662	2.3149	2.6758	2.0263
		60.0	46.477	13.4893	16.5694	11.7848	2.5287	3.0568	2.0263
		70.0	61.162	14.4206	18.7596	11.8689	2.6730	3.4047	2.0263
		80.0	76.812	14.9666	20.8293	11.9191	2.7564	3.7332	2.0263
		90.0	92.954	15.1462	22.8248	11.9358	2.7837	4.0526	2.0263
		100.0	109.095	14.9181	24.7391	11.9192	2.7484	4.3641	2.0263
		110.0	124.746	14.3785	26.7149	11.8690	2.6661	4.6936	2.0263
		120.0	139.431	13.4529	28.7423	11.7849	2.5227	5.0425	2.0263
		130.0	152.703	12.1075	30.8771	11.6664	2.3093	5.4238	2.0263
		140.0	164.160	10.2896	33.1909	11.5139	2.0108	5.8560	2.0263
		150.0	173.454	7.9290	35.7769	11.3305	1.6034	6.3652	2.0263
		160.0	180.302	4.9804	38.7223	11.1295	1.0570	6.9832	2.0263
		170.0	184.495	1.7374	41.8398	10.9548	0.3934	7.6893	2.0263
200.0	19.419	10.0	2.838	3.3668	3.4469	14.0081	0.6870	0.7019	2.3479
		20.0	11.264	8.2525	8.5858	14.3183	1.5638	1.6222	2.3479
		30.0	25.024	12.1140	12.8680	14.6249	2.1866	2.3131	2.3479
		40.0	43.699	15.0095	16.4375	14.8845	2.6228	2.8529	2.3479
		50.0	66.721	17.1783	19.5075	15.0921	2.9349	3.2998	2.3479
		60.0	93.391	18.7667	22.2332	15.2495	3.1564	3.6884	2.3479
		70.0	122.899	19.8060	24.6835	15.3598	3.2976	4.0346	2.3479
		80.0	154.347	20.4521	27.0313	15.4251	3.3850	4.3679	2.3479
		90.0	186.782	20.6672	29.2847	15.4467	3.4139	4.6916	2.3479
		100.0	219.216	20.4628	31.4946	15.4252	3.3867	5.0151	2.3479
		110.0	250.665	19.8368	33.7070	15.3601	3.3024	5.3475	2.3479
		120.0	280.173	18.7539	35.9884	15.2500	3.1545	5.7015	2.3479
		130.0	306.843	17.1712	38.4032	15.0928	2.9338	6.0907	2.3479
		140.0	329.865	14.9989	41.0569	14.8854	2.6213	6.5373	2.3479
		150.0	348.539	12.0980	44.1023	14.6261	2.1843	7.0764	2.3479
		160.0	362.299	8.2495	47.7849	14.3198	1.5633	7.7691	2.3479
		170.0	370.726	3.3614	52.3018	14.0099	0.6860	8.6890	2.3479
400.0	27.430	10.0	5.689	5.9814	5.9508	17.4320	1.0992	1.0992	2.6662
		20.0	22.582	12.5119	12.8749	17.9146	2.1333	2.1908	2.6662
		30.0	50.166	17.1787	18.0351	18.3368	2.7964	2.9254	2.6662
		40.0	87.602	20.5862	22.1433	18.6760	3.2507	3.4776	2.6662
		50.0	133.755	23.0547	25.5681	18.9399	3.5659	3.9226	2.6662
		60.0	187.220	24.9044	28.6191	19.1371	3.7964	4.3135	2.6662
		70.0	246.374	26.1175	31.4400	19.2741	3.9445	4.6734	2.6662
		80.0	309.420	26.8513	34.0362	19.3550	4.0333	5.0067	2.6662
		90.0	374.441	27.0952	36.5249	19.3822	4.0628	5.3307	2.6662
		100.0	439.462	26.8611	38.9668	19.3565	4.0348	5.6556	2.6662
		110.0	502.507	26.1477	41.4134	19.2771	3.9489	5.9901	2.6662
		120.0	561.661	24.9210	43.9322	19.1417	3.7994	6.3461	2.6662
		130.0	615.126	23.1190	46.6068	18.9461	3.5755	6.7387	2.6662
		140.0	661.279	20.6215	49.5678	18.6842	3.2562	7.1923	2.6662
		150.0	698.716	17.2281	53.0182	18.3474	2.8039	7.7467	2.6662
		160.0	726.300	12.5597	57.3392	17.9284	2.1409	8.4816	2.6662
		170.0	743.193	5.9664	63.2440	17.4501	1.0994	9.5657	2.6662

$$B_2 = \ln\left(\frac{1+\beta_3}{1-\beta_3}\right)\ln\frac{\omega_N}{\Delta E} + \ln\frac{\omega_f(1+\beta_3)}{m}\ln\left|\frac{m\,\omega - \omega_N\,\omega_f(1+\beta_3)}{m\,\omega_N - \omega_N\,\omega_f(1+\beta_3)}\right|$$
$$- \ln\frac{\omega_f(1-\beta_3)}{m}\ln\left|\frac{m\,\omega - \omega_N\,\omega_f(1-\beta_3)}{m\,\omega_N - \omega_N\,\omega_f(1-\beta_3)}\right|$$
$$+ L_2\left(\frac{m(\omega-\omega_N)}{m\,\omega - \omega_N\,\omega_f(1+\beta_3)}\right) - L_2\left(\frac{\omega_f(1+\beta_3)(\omega-\omega_N)}{m\,\omega - \omega_N\,\omega_f(1+\beta_3)}\right)$$
$$- L_2\left(\frac{m(\omega-\omega_N)}{m\,\omega - \omega_N\,\omega_f(1-\beta_3)}\right) + L_2\left(\frac{\omega_f(1-\beta_3)(\omega-\omega_N)}{m\,\omega - \omega_N\,\omega_f(1-\beta_3)}\right).$$

The Spence function, or dilogarithm is defined by

$$L_2(x) = -\int_0^x \frac{\ln|1-y|}{y}\,dy\,.$$

Its properties are extensively discussed in [13].

It is readily verified that for forward scattering ($t \to 0$), the value of δ approaches zero. The radiative corrections can become numerically important whenever $t > \mu^2 = .0195\ \text{GeV}^2$ provided the energy resolution ΔE is not too large.

The exact value of ΔE relevant for a given experiment depends on the experimental conditions. It is determined by the acceptances in angle and energy of the detector system and on the kinematical cuts made to define the reaction. It should thus be determined by the group performing the experiment. If the scattered particle is detected in a magnetic spectrometer, with very good angular resolution, then, if Δp_3 is the momentum resolution

$$\Delta E \cong \left[1 + \frac{\omega}{m}\left(1 - \frac{\beta_1}{\beta_3}\cos\theta_3\right)\right]\beta_3\,\Delta p_3\,.$$

If the two final particles are detected in coincidence with poor momentum resolution, the value of ΔE will be determined mainly by the angular acceptances. It will be very useful if future experimental results could be given with radiative corrections included, along with the value of ΔE used.

Description of the Tables

Table 1 (πN) and Table 2 (NN) give values of $100\,\delta_+$, $100\,\delta_-$, and $100\,\delta_0$, corresponding to the processes $\pi^+ p \to \pi^+ p$, $\pi^- p \to \pi^- p$ and $\pi^- p \to \pi^0 n$, respectively (for the nucleon-nucleon case the corresponding processes are $pp \to pp$, $\bar{p}p \to \bar{p}p$, $\bar{p}p \to \bar{n}n$), for various values of laboratory momentum (PLAB) and equally spaced values of center of mass scattering angle θ. The first column gives p_{lab} in GeV/c, the second column gives $W = \sqrt{s}$ = total center of mass energy in GeV, the third column gives θ in degrees, the fourth column gives the squared momentum transfer $|t|$ in $(\text{GeV/c})^2$, the fifth, sixth and seventh columns give $\delta_+(\Delta E_0)$, $\delta_-(\Delta E_0)$ and $\delta_0(\Delta E_0)$, all multiplied by 100, while the last three columns give values of the coefficients C_+, C_- and C_0 (also multiplied by 100) of $\ln(\Delta E_0/\Delta E)$ which occur in $\delta_+(\Delta E)$, $\delta_-(\Delta E)$ and $\delta_0(\Delta E)$, respectively. For a given value of ΔE we then have, for each process

$$\delta(\Delta E) = \delta(\Delta E_0) + C\ln(\Delta E_0/\Delta E)\,.$$

As mentioned previously, for $p_{lab} > 1$ GeV/c, the value of ΔE_0 was taken to be equal to the pion mass. For the case of scattering at lower energies, this value is rather unrealistic, as better energy resolution is always obtained. For this case, the value of ΔE_0 was taken to be one tenth of the incident particle's lab kinetic energy.

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